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DYNAMICS OF STOCHASTIC AND PERIODIC STRUCTURES IN MID-FREQUENCY RANGE

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ABSTRACT

Wave propagation in uncertain coupled structures is treated in this paper. The presented work is a hybridization of the Wave Finite Element Method (WFEM) with the Generalised Polynomial Chaos Expansion (GCE). The WFE is a spectral method dealing with wave propagation in periodic structure. This method proved its efficiency in different domains; structural vibration, non-destructive testing, etc. However, the WFE is limited to deterministic media. Knowing that uncertainties affect dynamic behavior in Mid- and High frequencies, the combination of WFE and GCE is used to predict the effect of uncertainties on the dynamic response of periodic media. The uses of the GCE is based on the iso-probabilistic transformations for usuel distributions to Gaussian one to use the Hermite-Chaos expansion.The presented approach is validated for two periodic waveguides connected through a junction with uncertain parameters. The obtained results are verified vs Monte Carlo simulations.

1 INTRODUCTION

Wave propagation in structures is of interest of many researchers in mechanical fields. There are many methods which allow the identification of wave characteristics such as Semi-Analytical Finite Element (SAFE) and Wave Finite Element method (WFE). The WFE is a spectral method dealing with wave propagation in structures with identical sub-systems (periodic waveguide)[1]. With a direct connection with the classical finite element method, the WFE can be simply developed. This study starts from the discretization of one sub-element, then a spectral eigenvalue problem is formulated regarding the periodicity of the structure. The WFE is then used for different problems; wave propagation [2], structural vibration, etc. If the structure contains some defects, the classical WFE is no longer applicable, so Mencik *et al.* developed a Diffusion Matrix Model (DMM) [2] which is based on the hybridization of the classical finite element method and the WFE to predict the reflection and transmission coefficients regarding these discontinuities. Knowing that uncertainties play a important role, this paper try to extend this formulation to non-deterministic structures. The use of the generalized chaos expansion allows the identification if uncertainty propagation in periodic media. This approach is based on the projection of stochastic parameters on the chaos base which allow, using non-intrusive approach, to quantify the statistics of outputs (kinematic and energetic parameters). The presented approach is then applied for two connected structures with uncertain inputs.

2 THE WAVE FINITE ELEMENT

This section deals with the dynamic behavior of a straight elastic dissipative structures, composed by N identical elements connected along the principal direction, say axis x, and whose left and right cross-sections are denoted as $\mathtt{L}$ and $\mathtt{R}$, respectively. The length of each subsystem is denoted as d. The formulation is based on the finite element model of a typical subsystem, and whose kinematic variables, displacements and forces, are written as $\mathbf{q}$ and forces $\mathbf{F}$ respectively. The dynamic equilibrium equation of on subsystem, at frequency $\omega/2\pi$, can be stated as follows:

$$\begin{pmatrix} \mathbf{D}_{\mathtt{LL}} & \mathbf{D}_{\mathtt{LR}} \\ \mathbf{D}_{\mathtt{RL}} & \mathbf{D}_{\mathtt{RR}} \end{pmatrix} \begin{pmatrix} \mathbf{q}_{\mathtt{L}} \\ \mathbf{q}_{\mathtt{R}} \end{pmatrix} = \begin{pmatrix} \mathbf{F}_{\mathtt{L}} \\ \mathbf{F}_{\mathtt{R}} \end{pmatrix} \tag{1}$$

where $(\mathtt{n} \times \mathtt{n})$ matrix $\mathbf{D}_{\mathtt{ij}} = \mathbf{K}_{\mathtt{ij}} - \omega^2 \mathbf{M}_{\mathtt{ij}}$ $(\{\mathtt{i}, \mathtt{j}\} \in \{\mathtt{L}, \mathtt{R}\})$ stands for the $\mathtt{ij}$ component of the dynamic stiffness operator condensed on the left and right cross-sections, namely $\mathbf{D}$ [1]. Here, $\mathbf{K}$ and $\mathbf{M}$ stand for the stiffness and mass matrices, respectively. Based on the periodicity of the structure and the Bloch's theorem, the spectral eigenvalue problem can be expressed as:

$$\mathbf{S}\mathbf{\Phi}_{\mathtt{i}} = \mu_{\mathtt{i}} \mathbf{\Phi}_{\mathtt{i}}. \tag{2}$$

This problem was established using the state vector representation. The resolution of equation (2) allows the identification of waves characteristics; propagation constant (μ_i) and deformed shape ($\mathbf{\Phi}_i$).

3 GENERALIZED POLYNOMIAL CHAOS EXPANSION

The polynomial chaos expansion (PCE), based on homogeneous chaos theory, was established by Wiener to describe stochastic processes using gaussian random variables. Ghanem and Spanos [4] used this formulation to describe the uncertainty propagation in structures. A second ordre stochastic process with finite variance can be expressed as:

$$u = \sum_{i=0}^{\infty} u_i \Psi_i(\xi) \overset{Order\ p}{\longrightarrow} u \approx \sum_{i=0}^{p} u_i \Psi_i(\xi) \tag{3}$$

This formulation is used for Gaussian distributions. Xiu and Karniadakis [5] extended it to non-Gaussian uncertainties by using the iso-probabilistic transformations (see table 1) which allow the use of Chaos-Hermite formulation.

Type of distribution	Transformation
Uniform (a, b)	$a + (b - a)\,(0.5 + 0.5erf(\xi/sqrt2))$
Normal (μ, σ)	$\mu + \sigma\xi$
Lognormal (μ, σ)	$exp(\mu + \sigma\xi)$

Table 1: Transformation of usuel distributions to Gaussian one

4 NUMERICAL VALIDATION

Let's define a structure composed by two identical stochastic and multi-modal waveguides connected through a coupling element (see Figure 1) with two different origin of coordinates. The length of two stochastic waveguides is $L = 0.5\ (m)$, the coupling elements' length is $0.01(m)$. The mechanical properties of studied structures are (see table 2): Poisson's ratio $\nu = 0.3$, dimension of cross section area $5 \cdot 10^{-2}\ m \times 3 \cdot 10^{-2}\ m$.

Variables	Type of distribution	Mean	Standard deviation
Young's Modulus	Lognormal	$2 \cdot 10^{11} Pa$	2%
Density	Lognormal	7800 kg/m^3	3%
Damping	Uniform	0.01	10%
Force	Normal	1 N	2%
Stiffness of coupling element	Uniform	$2 \cdot 10^{10} Pa$	2%

Table 2: Characteristics of studied waveguides

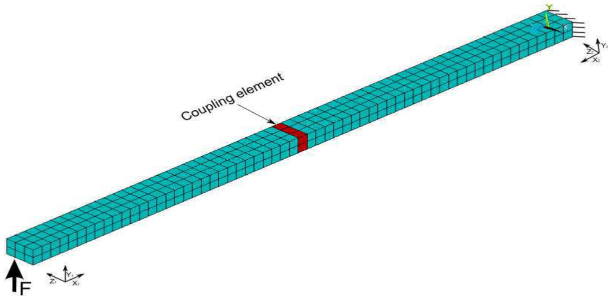

Figure 1: Two stochastic waveguides connected through a junction

The Figure 2 represents respectively the mean of reflection and transmission coefficients for different type of waves (1-2: flexural, 3; Torsional and 4; Longitudinal). The results of presented formulation (dashed line) are validated by Monte Carlo simulations.
The Figure 3 represents the mean of displacement calculed by spectral method and validated by MC simulations in $x_1 = 0.25$ m and $x_2 = 0.25$ m. The yellow area represents the envelope min-max of displacement regarding the stochastic input parameters.

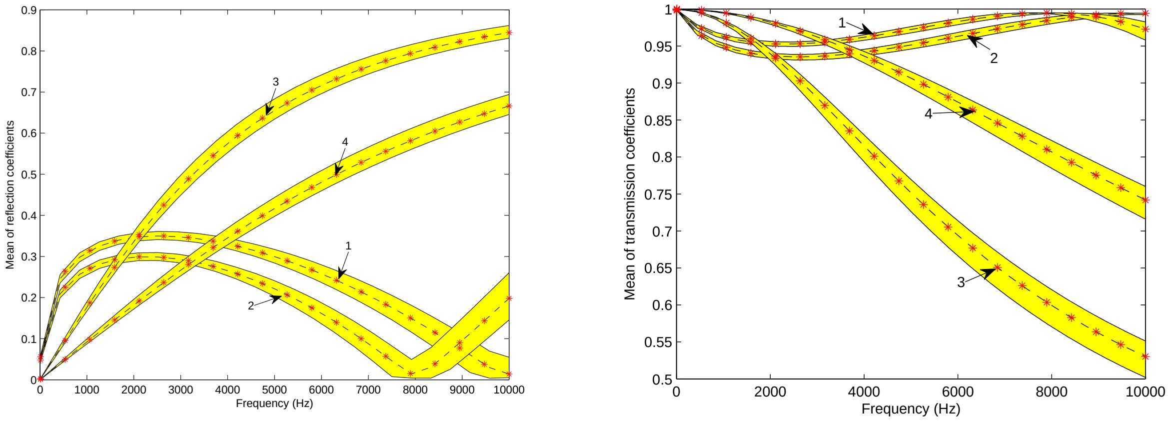

Figure 2: Mean of reflection (a) and transmission coefficients (b): (–) SWFE-Chaos, (*) Monte Carlo, yellow area: Interval min-max.

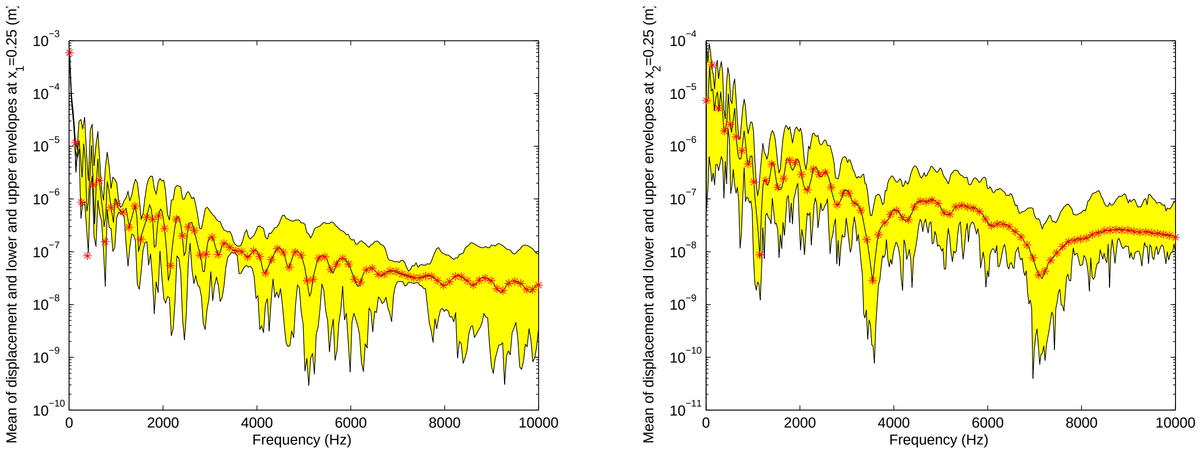

Figure 3: Mean of displacement at $x_1 = 0.25$ m (a)and $x_2 = 0.25$ m (b), (-) WFE-Chaos (4^{th} order expansion), (*) Monte Carlo.

REFERENCES

[1] W. X. Zhong and F. W. Williams. On the direct solution of wave propagation for repetitive structures. *Journal of Sound and Vibration*, 181:485–501, 1995.

[2] J. M. Mencik and M. N. Ichchou. Multi-mode propagation and diffusion in structures through finite elements. *European Journal of Mechanics - A/Solids*, 24(5):877–898, 2005.

[3] D. Duhamel, B.R. Mace, and M.J. Brennan. Finite element analysis of the vibrations of waveguides and periodic structures. *Journal of Sound and Vibration*, 294:205–220, 2006.

[4] R. Ghanem and P. Spanos. *Stochastic Finite Elements: A Spectral Approach*. Springer, New York, NY, 1991.

[5] D. Xiu and G. E. Karniadakis. Modeling uncertainty in steady state diffusion problems via generalized polynomial chaos. *Computer Methods in Applied Mechanics and Engineering*, 191:4927–4948, 2002.